which exhibits weak amplitude dependence, is $\simeq(1/n_2)$(orbital time). Thus, for a given satellite, the frequency of out-of-plane motion is fixed. On the other hand, pitching motion presents several interesting possibilities:

i) For satellites in circular orbits and with negligible solar radiation effect (i.e., $c_e = 0$), the motion of periodicity $\simeq (1/n_1)$ (orbital period) is assured.

ii) For initial conditions satisfying the relations

$$\left.\begin{aligned}\psi_0 &= \{c_e/(n_1{}^2-1)\}\sin(\theta_0-\alpha)\\ \psi_0' &= \{c_e/(n_1{}^2-1)\}\cos(\theta_0-\alpha)\end{aligned}\right\} \quad (5)$$

$D_1 = D_2 = 0$ and the pitching motion of orbital period ensues. The parameter $c_e/(n_1{}^2-1)$ appears to be quite significant as it governs not only the initial conditions leading to the periodic motion but also its amplitude. Thus, the initial conditions for the periodic motion are affected by solar radiation pressure in addition to orbital eccentricity and the satellite mass distribution. Interestingly, they can be represented quite simply by a circle as shown in Fig. 2.

iii) In general, when all the three terms are present, the motion is still periodic. However, the periodicity is now governed by K_1, which can be approximated as a rational number, m/n. As dictated by the third term, the lowest period is of course 2π, when $K_1 = m/n = 1$, the largest value for K_1 being one. Otherwise, the period would simply correspond to n orbits.

To assess accuracy of the above procedure, librational response of the system was obtained through numerical integration of the exact equations of motion (1a, 1b) using the initial conditions as given by Eq. (5) and compared with that predicted by the approximate solution of Eq. (4). Typical comparisons are presented in Fig. 3. The correlation is indeed satisfactory. Thus, the method represents a rather simple and effective procedure for determining periodic solutions of such a complex system. Its usefulness during initial stages of satellite design is apparent.

A comment concerning the applicability of this procedure to more complex situations involving spacecraft flexibility and/or damping would be appropriate. Periodic solutions continue to be important[11] as their spinal character with respect to the stability region is maintained.[12] Of course, routine algebraic manipulations would increase because of the rather involved character of integrand f_i*. However, the method should provide a quick estimate of the periodic solutions, even for such formidable systems.

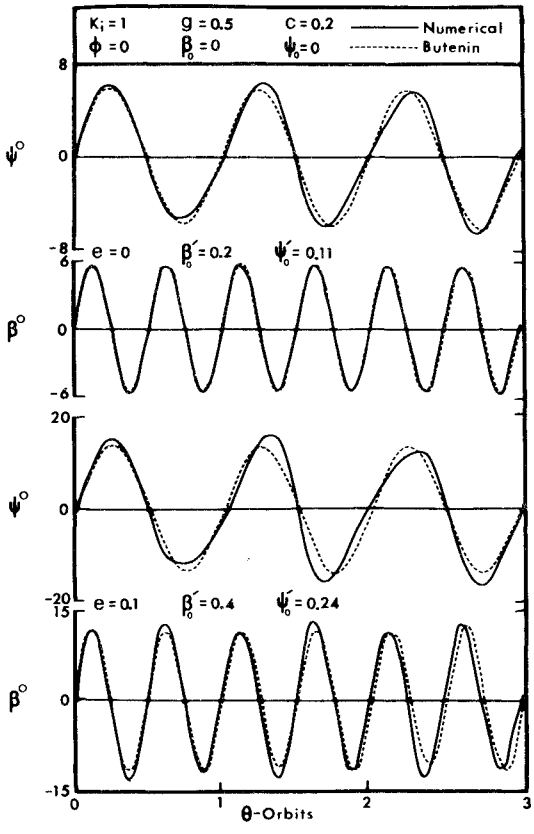

Fig. 3 Typical examples of approximate periodic librations.

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Studies on Mixed Fuels—Hydrazine and Ethyl Alcohol System

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Introduction

ALTHOUGH hydrazine has long been known to be an attractive rocket fuel, several problems arise in its use. Hydrazine readily undergoes catalytic decomposition on many metal surfaces. In the vapor phase it has a strong tendency towards explosive decomposition. Hence for storing hydrazine

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Table 1 Physical properties of the mixed fuel

Mole % of N_2H_4 in C_2H_5OH	density g/cm³ 25°C	density g/cm³ 40°C	viscosity Centipoise 25°C	viscosity Centipoise 40°C	surface tension dynes/cm² 25°C	surface tension dynes/cm² 40°C
0.00	0.7853	0.7734	1.159	0.811	21.820	20.658
37.99	0.8654	0.8539	1.553	1.028	24.796	24.078
55.04	0.8958	0.8875	1.648	1.099	26.999	25.416
60.81	0.9229	0.9126	1.665	1.136	28.082	27.146
69.54	0.9404	0.9322	1.564	1.192	29.913	29.299
77.20	0.9489	0.9420	1.197	0.924	32.272	31.251
84.56	0.9698	0.9587	1.102	0.898	34.955	34.191
91.11	0.9831	0.9749	1.006	0.819	38.628	40.184
94.48	0.9948	0.9851	1.001	0.812	43.450	43.669
100.00	1.0026	0.9956	0.894	0.727	65.945	53.165

special types of containers have to be provided. From the viewpoint of storability, it is desirable to use mixed fuels with hydrazine as one component. In many developing countries, hydrazine is not as inexpensive as in the U.S.A. Accordingly, a hydrazine and ethyl alcohol mixture has been chosen for detailed study regarding its potentiality. Ethyl alcohol was chosen because of its low cost and availibility. It should be noted that hydrazine and ethyl alcohol/N_2O_4 system has been investigated by earlier workers.[1]

Experimental

Materials

Red fuming nitric acid, RFNA (Basic and Synthetic Chemicals, Calcutta) containing 2 weight percent NO_2 was used. Anhydrous hydrazine was prepared from hydrazine hydrate as described previously.[2] Absolute alcohol was purified in the usual manner.[3] Hydrazinium nitrate was prepared by the reaction between barium nitrate and hydrazine sulphate.[4]

Procedure

a) Determination of physical properties

Density was measured by pycnometry. Viscosity was measured by Ostwald's viscometer whereas the surface tension was measured by capillary rise. All the measurements were made in a constant temperature thermostat whose temperature was maintained correct to ± 0.01°C. The results are recorded in Table 1.

b) Determination of hydrazine concentration

Hydrazine concentration was estimated by iodate method.[5] In order to investigate the relative storability of hydrazine and mixtures of hydrazine and ethyl alcohol, the hydrazine concentration in the two types of fuels kept in glass containers was estimated for more than a year on several days. The results are summarized in Table 2.

c) Ignition delay measurements

Ignition delay of the mixed fuel was initially measured by cup-test method but later on the experiments were repeated using photo-cell and oscilloscope technique as described earlier.[6] The final results are given in Table 3.

Table 2 Stability data of hydrazine and mixed fuel containing 79% and 63% by weight of hydrazine

Hydrazine Time days	Hydrazine Concentration % wt	Hydrazine+ethanol Time days	Hydrazine+ethanol Concentration % wt	Hydrazine+ethanol Time days	Hydrazine+ethanol Concentration % wt
0	101±1	0	79±1	0	63±1
2	100±1	6	78±1	70	63±1
4	100±1	18	77±1	365	63±1
11	99±1	70	76±1	477	63±1

Table 3 Effect of hydrazinium nitrate on ignition delay of (hydrazine+ethanol)/RFNA system

N_2H_4 (weight %)	Ignition delay (msec) No ignition catalyst	Ignition delay (msec) Hydrazinium nitrate as catalyst (5% by wt in fuel)
100	30±15	>20
82	60±20	30±10
65	184±45	60±20
45	does not ignite	50±26
24	does not ignite	does not ignite

d) Analysis of gaseous products

Qualitative analysis of the combustion products of hydrazine/RFNA and hydrazine-ethyl alcohol/RFNA system was made by using an Orsat gas analyser.

e) Calculation of performance parameters[7]

The performance parameters viz., chamber temperature, Tc; specific impulse, I_{sp}; and characteristic velocity, c^* were calculated assuming frozen flow criterion. The results are recorded in Tables 4 and 5.

Discussion

Data recorded in Table 2 show that mixed fuel containing 63 weight percent hydrazine is stable in glass containers even for a year, whereas hydrazine as such decomposes appreciably in ten days. It seems that lower the hydrazine concentration, greater the stability. At first instance, it would appear that this stability is imparted because of a complex formation between hydrazine and ethyl alcohol. However, qualitative measurements show that heat of mixing is negative, whereas volume of mixing is positive. The later observation does not support the complex formation hypothesis. The plot of density vs composition yields a straight line which shows that probably complex formation does not take place. However, it should be noted that both hydrazine and ethyl alcohol have a tendency for self-association and the self-association has to be broken before complexation takes place. Hence, the aforementioned experimental results

Table 4 Performance parameters of mixed fuels of varying composition[a]

Hydrazine (% wt)	I_{sp} (sec)	Tc (°K)	c^* (m/sec)
100	236.0	2899	1656.50
90	234.0	2845	1644.35
80	230.5	2743	1620.31
70	225.1	2599	1583.24
60	220.6	2482	1533.71
40	210.6	2188	1469.81

[a] RFNA (6% NO_2 by wt), O/F = 1.5, Pc/Pe = 20 (Pe = ambient pressure).

Table 5 Variation of specific impulse (I_{sp}) with mixture ratio (O/F)[a]

O/F	Specific impulse (sec)
1.0	212
1.5	225
2.0	224
2.5	206
3.0	197

can not be taken as unambiguous evidence against the complex formation hypothesis.

For a good fuel the requirements are a) high density; b) low viscosity; and c) low surface tension.[8] Comparing the results for hydrazine and mixed fuel given in Table 1, it can be seen that mixed fuel is no better with respect to a and b, however, it is a better choice from the viewpoint of c.

Data recorded in Table 3 show that the ignition delay is longer in the case of mixed fuel as compared to hydrazine. This is expected since ethyl alcohol partly inhibits the decomposition of hydrazine. It is also observed that ignition delay increases with the increase of ethyl alcohol concentration. However, the delay can be reduced by the addition of hydrazinium nitrate in the fuel. Thus, for 82 weight percent hydrazine it can be reduced to 30 msec on adding hydrazinium nitrate.

The qualitative analysis of gaseous combustion products indicated the presence of N_2O, N_3H, NH_3 and N_2. When the mixed fuel is used the only additional product is aldehyde produced by the oxidation of alcohol. This means that during the combustion of mixed fuels simultaneous reactions take place.

The values of performance parameters recorded in Table 4 show that I_{sp}, Tc, and c^* all decrease with decrease in hydrazine content. The storability of the mixed fuel increases with the decrease in the concentration of hydrazine. For practical purposes, optimization would be necessary and judged from all angles mixed fuel containing 70–80%, by weight of hydrazine should be good enough. Table 5 shows the variation of specific impulse with mixture ratio for a typical composition of the mixed fuel. The results show that mixture ratio = 1.5 is the best.

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Average Structural Response to Locally Stationary Random Excitation

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Introduction

IN physical problems, the concept of a locally stationary process can be used to approximate nonstationary excitations when the average instantaneous power is varying slowly with respect to memory time.[1] Using this concept, it is sometimes possible to break up a nonstationary excitation into a number of smaller samples that exhibit weakly stationary properties.

The response of a time-invariant linear system to this type of nonstationary excitation follows from well known principles. Analogous, in the probabilistic sense, to the transient (nonstationary) and steady-state (weakly stationary) response to a stationary excitation,[2] there is a transient and average (weakly stationary) response to a nonstationary excitation.[3] The transient response of a time-invariant linear system to a locally stationary excitation is discussed in Ref. 4. Of interest here is the average response.

It is noted that the average response to a locally stationary excitation is of interest because it plays an important role in some design studies. In aeronautical applications, for example, this concept is used to approximate the nonstationary atmospheric turbulence phenomenon, and military design specifications for aircraft have been developed using this approach.

In the present Note, the average properties of the excitation are determined by applying a time version of the usual treatment of a sequence of random variables. This approach corresponds to the singular case considered in Ref. 3 and appears to be alluded to in Ref. 5. For a stationary process, this technique leads to the usual theorems on ergodicity. The average response properties follow by considering the average excitation as a weakly stationary input.

General Concepts

In the present context, locally stationary means that a long record $x(t)$ of length T can be separated into a sequence of distinct independent phases ${}^jx(t), j = 1,\ldots,N$, where each phase lasts for a time ΔT_j which is long enough to exhibit dominantly weakly stationary properties. That is

$$x(t) = {}^jx(t) \quad \text{on} \quad (T_{j-1}, T_j], \qquad j = 1,\ldots,N \tag{1}$$

where

$$T = \sum_{j=1}^{N} \Delta T_j \tag{2}$$

and

$$\Delta T_j = T_j - T_{j-1}, \qquad j = 1,\ldots,N \tag{3}$$

If each phase is described by its mean ${}^j\mu_x$, its autocorrelation ${}^jR_x(\tau)$, and its distribution ${}^jF_x(x)$, then these quantities are given by the expected values

$${}^j\mu_x = E[{}^jx(t)] \tag{4}$$

$${}^jR_x(\tau) = E[{}^jx(t+\tau){}^jx(t)] \tag{5}$$

$${}^jF_x(x) = E[{}^jz(t)] \tag{6}$$

where ${}^jz(t)$ is the process

$${}^jz(t) = \begin{cases} 1, & \text{if } {}^jx(t) \leqq x \\ 0, & \text{if } {}^jx(t) > x \end{cases} \tag{7}$$

and x is an arbitrary but fixed constant.

One approach to determining the average statistical properties of $x(t)$ is to formulate the over-all properties of the sequence of time dependent random variables ${}^jx(t), j = 1,\ldots,N$. To illustrate, consider the time average

$$\bar{x}(t) = \frac{1}{T}\int_0^T x(t)\,dt \tag{8}$$

of the given process $x(t)$. Clearly, from Eq. (1),

$$\bar{x}(t) = \frac{1}{T}\sum_{j=1}^{N}\int_{T_{j-1}}^{T_j} {}^jx(t)\,dt \tag{9}$$

where $\bar{x}(t)$ is a random variable.

Equation (9) is a time version of the estimate of the sample mean for a sequence of random variables. Therefore, the over-all mean is the expected value of Eq. (9). That is,

$$\overline{E[x(t)]} = \frac{1}{T}\sum_{j=1}^{N}\int_{T_{j-1}}^{T_j} E[{}^jx(t)]\,dt \tag{10}$$

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